Nataliya Zubko et al.: Analysis of VLBI Data with Different Stochastic Models, IVS 2012 General Meeting Proceedings, p.236–240
http://ivscc.gsfc.nasa.gov/publications/gm2012/Zubko.pdf

Analysis of VLBI Data with Different Stochastic Models

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Abstract

VLBI observations are generally analyzed using the least-squares method. For accurate results the functional and stochastic models need to be well-defined. In the standard stochastic model the variance-covariance matrix is dependent on only one stochastic parameter, described by common level of variance. The analysis of observations can be improved by taking into account additional parameters in the stochastic model, such as station and elevation angle dependent effects. Thus the model becomes reliant on several stochastic properties. A stochastic model, which includes the station and the elevation angle dependence of observations, has been implemented in the VieVS software. We present results of a comparative analysis using a traditional and an advanced stochastic model. In the advanced stochastic model the variance components of VLBI observations were estimated with the MINQUE method.

1. Introduction

Various parameters can be estimated from VLBI observations, such as the Earth orientation parameters, station coordinates, troposphere delays and others. In the analysis of VLBI data the least-squares method is frequently used. Since VLBI measurements are dependent on various random and systematic errors, it is important to choose an appropriate model for the system. In order to obtain accurate results, it is desirable to incorporate all these errors into a stochastic model that is described with the variance-covariance matrix.

Within the stochastic model normally used in VLBI data analysis, the observations are assumed to be uncorrelated and to have the same variance. This assumption simplifies the analysis of VLBI data. However, it affects the accuracy. The correlation between VLBI observations was studied, for example, in [1] and [2], and it was shown that the results can be improved by using a full variance-covariance matrix. In [3] a refined stochastic model for the analysis of VLBI data was proposed, where, instead of a common variance, several variance components are included into the variance-covariance matrix.

In the advanced stochastic model the variance-covariance matrix is constructed so that it depends on a number of variance components. Since the correlation process reveals strong station and elevation angle dependencies, we have included the station and elevation angle dependent variance components in our stochastic model. For estimation of variances, we apply the Minimum Norm Quadratic Unbiased Estimation (MINQUE), developed by [4]. This method is often used for the estimation of variance components in different applications, for example, in VLBI analysis by [5] and [3] and in GPS analysis by [6] and [7]. The advanced stochastic model and the MINQUE algorithm were implemented in the VieVS software [8]. We have investigated the variance components in the variance-covariance matrix of the advanced stochastic model. Here we present results of comparison of estimated station coordinates and polar motion obtained with two stochastic models.

2. Stochastic Models

In order to estimate the unknown parameters, we use the Gauss-Markoff model with two different stochastic properties. In the first adjustment the covariance matrix is chosen as follows:

$$D(y) = \sigma^2 Q = \sigma^2 P^{-1}, \tag{1}$$

where σ^2 is a factor describing the variance level of the observations and Q and P are the cofactor and the weight matrices, respectively. The correlations between observations were neglected. Thus the cofactor matrix Q is a diagonal matrix with the elements $\sigma_i^2 + \sigma_{const}^2$, where σ_i is a formal error derived from the correlation process for the i^{th} observation and σ_{const} is added to cover the deficiency of the model. It is usually assumed to be equal to 1 cm^2. The variance factor in (1) is estimated with:

$$\hat{\sigma}^2 = \frac{\hat{e}P\hat{e}}{n-u}. \tag{2}$$

In the second adjustment, the covariance matrix takes the form:

$$D(y) = \sum_{m=1}^{m_{max}} \sigma_m^2 V_m, \tag{3}$$

where $\sigma_m^2 (m = 1, ..., k)$ are variance components and V_m $(m = 1, ..., k)$ are accompanying matrices. It is assumed that the accompanying matrices V_m are known and the variance components σ_m^2 need to be estimated, for example, with the MINQUE method. In the second model, the correlations between observations are also neglected. Therefore, the accompanying matrices V_m are diagonal ones.

Our model includes the common level of variance σ_{com}^2, the additive variance σ_{add}^2, antenna dependent variances $(\hat{\sigma}_{t1}^n)^2$ and $(\hat{\sigma}_{t2}^n)^2$ for the pair of telescopes per observation, and antenna elevation dependent variances $(\hat{\sigma}_{el1}^m)^2$ and $(\hat{\sigma}_{el2}^m)^2$ for the pair of elevation angles. The variance-covariance matrix can be expressed as follows:

$$D(y) = \hat{\sigma}_{com}^2 V_{com} + \hat{\sigma}_{add}^2 V_{add} + \sum_{n=1}^{n_{max}} (\hat{\sigma}_{t1}^n)^2 V_{t1}^n + \sum_{n=1}^{n_{max}} (\hat{\sigma}_{t2}^n)^2 V_{t2}^n + \sum_{m=1}^{m_{max}} (\hat{\sigma}_{el1}^m)^2 V_{el1}^m + \sum_{m=1}^{m_{max}} (\hat{\sigma}_{el2}^m)^2 V_{el2}^m, \tag{4}$$

where n_{max} is the number of telescopes participating in the observations and m_{max} is a number of gradations of telescope elevation angles.

The elements of V_{com} are formal errors derived from the correlation process and V_{add} is the identity matrix. The elements of the V_{t1} and V_{t2} matrices are filled as follows: if in the i^{th} observation the telescopes with order numbers k and l have been observing, the corresponding elements of matrices V_{t1}^k and V_{t2}^l are filled with a formal error derived from the correlator, whereas the elements for all other V_{t1}^n $(n \neq k)$ and V_{t2}^n $(n \neq l)$ corresponding to the i^{th} observation are equal to zero. The accompanying matrices V_{ell} are filled in the same way.

3. Results

We analyzed approximately 50 VLBI sessions observed in the year 2010 with the two stochastic models. The variance components for the advanced stochastic model were obtained. Table 1 contains the mean values of estimated variances with the iterative MINQUE method and their standard deviations. It is important to note that for each station the number of sessions is different. Namely, Zelenchukskaya participated in only 10 sessions, whereas Wettzell participated in 30 sessions. Variances in this estimation can take negative values; however, the covariance matrix (3) must be positive. An estimation of the variances requires 5-15 iterations. Note, the estimation was impossible for some sessions due to divergence of the iterative process. Therefore within the advanced stochastic model, some sessions cannot be processed.

Table 1. Mean values of estimated variance components of the advanced stochastic model.

Type of variance	**Mean value**	**Standard deviation**
Common	0.5656	0.3755
Additive	0.3728	0.2074
Wettzell	0.4789	0.4839
Matera	0.3497	0.718
Kokee	0.651	0.6807
Badary	0.7398	0.6957
Zelenchk	1.016	0.8325
Onsala60	0.4645	0.5739
Nyales20	0.03644	0.379
Westford	0.4498	0.4761
Hobart26	1.577	1.592
El. angle=$5^\circ - 8^\circ$	0.6513	0.9669
El. angle=$8^\circ - 11^\circ$	0.4773	0.6352
El. angle=$11^\circ - 15^\circ$	0.3354	0.6252
El. angle=$15^\circ - 20^\circ$	0.1962	0.4834
El. angle=$20^\circ - 30^\circ$	0.01549	0.4829
El. angle=$30^\circ - 45^\circ$	0.004851	0.3911
El. angle=$45^\circ - 65^\circ$	-0.03137	0.4461

Figure 1 presents the repeatability of the X, Y, and Z coordinates of a few antennas. As one can see, there is no large difference between the rms values of the station coordinates. The rms values calculated from the results, where the advanced stochastic model was applied, is lower for some stations.

Figure 2 shows residuals of polar motion xpol and ypol coordinates. The rms values of the polar motion residuals are also lower if the advanced stochastic model is used. The difference between the rms values for the xpol residuals is 0.017 mas, and for ypol it is 0.011 mas.

4. Conclusions

In the analysis of VLBI data the stochastic model with a common variance factor is frequently used. The stochastic model with several variance components can slightly improve the results. Small differences between the two approaches confirm the reliability of the traditional stochastic model; however, for the achievement of high accuracy, more attention should be given to the model.

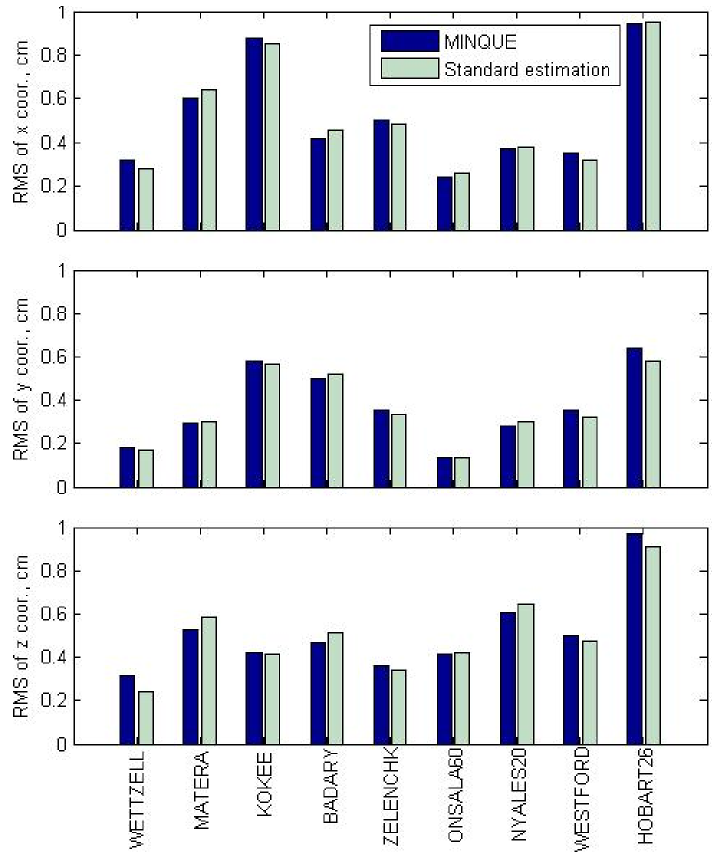

Figure 1. Repeatability of station coordinates.

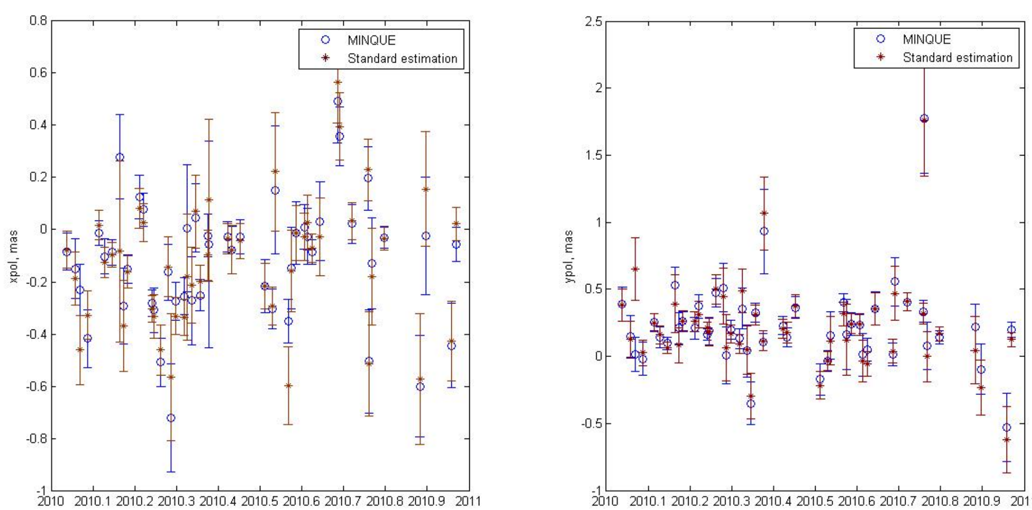

Figure 2. Comparison of polar motion residuals.

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